

Fall Interschool 2010 Answer Key

1. 3
2. 7
3. 6
4. 5
5. $3i$
6. 3
7. 4
8. 7.5
9. 13824

For question 10, I mistakenly forgot to ask for a rounded answer. If you gave a rounded answer, then you will receive half credit. If you gave an exact answer, then you will receive full credit. Yes, I acknowledge that these answers are ridiculous.

10a.

$$\text{Let } \theta_1 = \arctan \left(\frac{5 - \sqrt{25 - 4 \left(\frac{4.9 \cdot 25}{3600} \right) \left(100 + \frac{4.9 \cdot 25}{3600} \right)}}{\frac{2 \cdot 4.9 \cdot 25}{3600}} \right) \text{ and}$$

$$\text{let } \theta_2 = \arctan \left(\frac{15 + \sqrt{225 - 4 \left(\frac{4.9 \cdot 225}{3600} \right) \left(100 + \frac{4.9 \cdot 225}{3600} \right)}}{\frac{2 \cdot 4.9 \cdot 225}{3600}} \right)$$

Then $x + y = \frac{3600 \sin(2\theta_1)}{9.8} + \frac{3600 \sin(2\theta_2)}{9.8}$ or equivalent, which is about 48.611.

$$10b. z = \frac{\tan(\theta_1) + \sqrt{\tan^2(\theta_1) - 4 \left(\frac{4.9(\tan^2(\theta_1) + 1)}{3600} \right) (100)}}{\frac{9.8(\tan^2(\theta_1) + 1)}{3600}} \text{ or equivalent, which is about 25.6976.}$$

11. $\arcsin(2 \sin(10) \sin^2(110)) - 10$, $70 - \arcsin(2 \sin(10) \sin^2(110))$, $10 + \arcsin(2 \sin(10))$, $110 - \arcsin(2 \sin(10))$ or equivalents.

12. 11118888

13. 79

14. 58

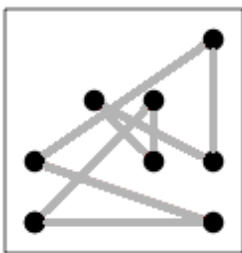
15. 5

16. $\frac{3}{16} - \pi/32$

17. 1, 6, powers of 2, and primes.

18. $\frac{7\sqrt{29}}{10} + \frac{2}{5} \sinh^{-1}\left(\frac{5}{2}\right) + \frac{5}{2} \sinh^{-1}\left(\frac{2}{5}\right)$ or equivalent.

19. 3
 20a. Plane missing a circle
 20b. Hyperboloid
 21. 0.394
 22. God created the integers. All else is the work of man. -Leopold Kronecker
 23. 500001
 24. $p_n(1) = \frac{i^{n+1}}{n!} \sum_{k=1}^{n+1} \sum_{j=0}^k \binom{k}{j} \left(\frac{(-1)^j (k-2j)^{n+1}}{2^k i^k k} \right)$, where $i = \sqrt{-1}$.
 25. 1250541700
 26. Every other fibonacci number (1, 3, 8, 21...)
 27a. $x^3 - x^2 - 8x + 4$
 27b. $3x^3 - 5x^2 - 6x - 7$
 28. $(-5, -4) \cup (-3, -1) \cup (-1, 1] \cup [2, 3] \cup \{5\}$
 29. 0.266
 30a. 100
 30b. 611111
 3i. $4/3$
 32. 42787 or 65337 (only one is necessary, but both will be accepted)
 33. See below
 34. $\arctan(10/9.8)$
 35. $\sqrt{784/75}$
 36. N = 3, E = 9, P = 4, T = 7, U = 5, S = 1, A = 2, R = 0, L = 6, O = 8
 37. 2 2 2 3 4 5 7 8 9 10 with no 5 cards the same suit. Yields sour hand of 10 high.
 38. 210
 39. 242.802
 40. $\tan\left(\frac{n+1}{2} x\right)$ or other reasonable simplifications
 41a. Yes
 41b. No
 42. 24 and 42
 43. 193
 44. $\binom{n+1}{2}$
 45. $4c/5$
 46. $\frac{6+3\sqrt{3}}{2}$ 62 linear inches
 47. (103, 78), (103, 272), (713, 272)
 48. $p = 377, q = -610$
 49. $\sqrt{2}/2$
 50.



51. 24
 52. Draw
 33.

15+ 6	14+ 2	2- 5	3	3- 1	4
4	3	10x 2	5	5- 6	1
5	6	3	1	15+ 4	1- 2
7+ 2	4	1	6	5	3
3÷ 3	4- 1	2- 6	4	1- 2	1- 5
1	5	2÷ 4	2	3	6

Note: If you have disputes, then email the grader at InterschoolDisputesFall2010@hotmail.com. This includes those of you who left disputes in the form of comments on the answer sheet. Email the grader just to be sure.