Fall Interschool 2010 Answer Key

1. 3
2. 7
3. 6
4. 5
5. $3i$
6. 3
7. 4
8. 7.5
9. 13824

For question 10, I mistakenly forgot to ask for a rounded answer. If you gave a rounded answer, then you will receive half credit. If you gave an exact answer, then you will receive full credit. Yes, I acknowledge that these answers are ridiculous.

10a.

Let $\theta_1 = \arctan\left(\dfrac{5-\sqrt{25-4\left(\frac{4.9\cdot 25}{3600}\right)\left(100+\frac{4.9\cdot 25}{3600}\right)}}{\frac{2\cdot 4.9\cdot 25}{3600}}\right)$ and

let $\theta_2 = \arctan\left(\dfrac{15+\sqrt{225-4\left(\frac{4.9\cdot 225}{3600}\right)\left(100+\frac{4.9\cdot 225}{3600}\right)}}{\frac{2\cdot 4.9\cdot 225}{3600}}\right)$

Then $x + y = \dfrac{3600\sin(2\theta_1)}{9.8} + \dfrac{3600\sin(2\theta_2)}{9.8}$ or equivalent, which is about 48.611.

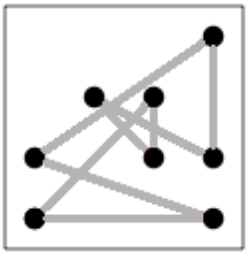
10b. $Z = \dfrac{\tan(\theta_1)+\sqrt{\tan^2(\theta_1)-4\left(\frac{4.9(\tan^2(\theta_1)+1)}{3600}\right)(100)}}{\frac{9.8(\tan^2(\theta_1)+1)}{3600}}$ - 5 or equivalent, which is about 20.6976.

11. $\arcsin(2\sin(10)\sin^2(110))-10$, $70-\arcsin(2\sin(10)\sin^2(110))$, $10+\arcsin(2\sin(10))$, $110-\arcsin(2\sin(10))$ or equivalents. Question has been thrown out.

12. 11118888
13. 79
14. 58
15. 5
16. 3/16 – pi/32
17. 6, powers of 2, and primes. 1 has been removed from the answer, as it yields the empty set.

18 has been changed to. $\dfrac{\sqrt{29}}{2}+\dfrac{2}{7}\sinh^{-1}\left(\dfrac{5}{2}\right)+\dfrac{25}{14}\sinh^{-1}\left(\dfrac{2}{5}\right)$ or equivalent.

19. 3
20a. Plane missing a circle. Circle is also acceptable.
20b. Hyperboloid
21. 0.394
22. God created the integers. All else is the work of man. –Leopold Kronecker
23. 500001
24. $p_n(1) = \frac{i^{n+1}}{n!}\sum_{k=1}^{n+1}\sum_{j=0}^{k}\binom{k}{j}\left(\frac{(-1)^j(k-2j)^{n+1}}{2^k i^k k}\right)$, where $i = \sqrt{-1}$.
25. 1250541700
26. Every other fibonacci number (1, 3, 8, 21...). Question has been thrown out.
27a. $x^3 - x^2 - 8x + 4$
27b. $3x^3 - 5x^2 - 6x - 7$
28. $(-5,-4) \cup (-3,-1) \cup (-1,1] \cup [2,3] \cup \{5\}$
29. 0.266
30a. 100
30b. 611111
3*i*. 4/3
32. 42787 or 65337 (only one is necessary, but both will be accepted)
33. See below
34. arctan(10/9.8)
35. sqrt(784/75)
36. N = 3, E = 9, P = 4, T = 7, U = 5, S = 1, A = 2, R = 0, L = 6, O = 8
37. 2 2 2 3 4 5 7 8 9 10 with no 5 cards the same suit is an acceptable answer and there are others. Any set of 10 cards that yields a sour hand of 10 9 8 7 5 will be accepted.
38. 210
39. 242.802
40. $\tan\left(\frac{n+1}{2}x\right)$ or other reasonable simplifications
41a. Yes
41b. No
42. 24 and 42
43. 193
44. $\binom{n+1}{2}$
45. 4*c*/5
46. $\frac{6+3\sqrt{3}}{2}$
47. (103, 39), (713, 272). Answer (103, 272) has been removed.
48. $p = 377$, $q = -610$
49. sqrt(2)/2



50.
51. 24
52. Draw. NOTE: This answer is the same no matter how the board is read with regards to White moving bottom to top or top to bottom. See http://en.wikipedia.org/wiki/Richard_Reti
33.

15+ 6	14+ 2	2 – 5	3	3 – 1	4
4	3	10 × 2	5	5 – 6	1
5	6	3	1	15+ 4	1 – 2
7+ 2	4	1	6	5	3
3 ÷ 3	4 – 1	2 – 6	4	1 – 2	1 – 5
1	5	2 ÷ 4	2	3	6

Note: If you have disputes, then email the grader at InterschoolDisputesFall2010@hotmail.com. This includes those of you who left disputes in the form of comments on the answer sheet. Email the grader just to be sure.